Volume-2, Issue-5, September-2018: 49-51
International Journal of Recent Innovations in Academic Research ISSN: 2635-3040

Formulation of a Class of Standard Cubic Congruence Modulo A Positive Integer-Multiple of Nine

Prof. Roy, B.M.

Head, Department of Mathematics, Jagat Arts, Commerce & I H P Science College, Goregaon Dist. Gondia, Maharastra State, India.
Affiliated to RTM Nagpur University, Nagpur, Pin-441801
Corresponding Author E-mail: roybm62@gmail.com

Abstract: In this paper, a class of standard cubic congruence of modulus positive- integer multiple of nine is formulated. Formulation of solutions is the merit of the paper. The formula is tested true by solving suitable examples. No Previous formulation was found in the literature. First time a formulation is established.
Keywords: Cubic Residues, Formulation of Solutions, Standard cubic-congruence.

Citation: Roy, B.M. 2018. Formulation of a Class of Standard Cubic Congruence Modulo A Positive Integer-Multiple of Nine. International Journal of Recent Innovations in Academic Research, 2(5): 49-51.
Copyright: Roy, B.M., **Copyright©2018.** This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Introduction

Congruence is called the heart of Number Theory and it is an important branch of Mathematics. It is said that Number theory is pivoting the modern industrial development. Such is the importance of this theory. Even a very little is done on it. Cubic congruence is a part of it. But nothing is found in the literature.

A congruence of the type $x^3 \equiv a \ (mod\ m)$ is called a standard cubic congruence. If a is cubic residue of m, then the congruence is called solvable. If m is prime or composite, then the congruence is called of prime or composite modulus. Every residue is a solution of a cubic congruence. The cube of a residue is called a cubic-residue$i.e.$ if r is a residue of m and $r^3 \equiv a(mod\ m\ ), then\ a\ is$ called a cubic residue of m.

Some cubic congruence has a unique solution; some has exactly three solutions while some other has more than three solutions. Here, a class of standard cubic congruence having exactly three solutions is under consideration.

Literature Review

Much had been written on quadratic congruence but nothing is found in the literature of mathematics on cubic congruence. I have formulated many standard quadratic congruence of prime and composite modulus. I had gone through many books on Number Theory and found many materials about quadratic congruence but only a definition of standard cubic congruence (Thomos Koshy, 2007). No one cared the cubic congruence.

Need of Research

A cubic Congruence is an essential part of congruence in number theory and I tried my best to improve the theory by formulating the solutions. Formulation of solution is the main aim of this study. No formulation was found in the literature. But the use of Chinese Remainder Theorem. To establish a formula is the need of my research.

Problem-Statement

To formulate the solutions of standard cubic congruence of the type $x^3 \equiv b^3 \ (mod\ 9m);\ m\ being\ any\ positive\ integer.$

The formula for all the solutions of it would be proved to be as:

$x \equiv 3mk + b\ (mod\ 9m),\ \ m\ \ being\ any\ positive\ integer\ with\ k = 0, 1, 2.$

Analysis and Result (Formulation)

Consider the cubic congruence of the type

$x^3 \equiv b^3\ (mod\ 9m);\ m\ being\ a\ positive\ integer.$

For $x \equiv 3mk + b\ (mod\ 9m)$,

It can be seen that $x^3 = (3mk + b)^3$

$$= (3mk)^3 + 3.(3mk)^2.b + 3.3mk.b^2 + b^3$$
$$= 9mk(\ldots\ldots) + b^3$$
$$\equiv b^3\ (mod\ 9m)$$

Thus, we see that $x \equiv 9mk + b$ satisfies the standard cubic congruence under consideration. Hence, it is a solution of it for some positive integer k $i.e.k = 0,1,2,3, \ldots$ ....

For $k = 0, 1, 2$ we get three different values of x as solutions of the congruence.

But for $k = 3, we\ get$ the same solution as for k=0; also for k=4 we get the same solution as for k=1; for k=5, the solution is the same for k=2; also for k=6, we get the same solution as for k=0. Thus we conclude that there are three different solutions of the congruence under consideration.

We also know that every residue of 9m is a solution of the cubic congruence (Zuckerman *et al.*, 2008) and every congruence has exactly three solutions; hence there must be $\frac{1}{3}(9m) = 3m$ congruence.

Formulation

In the conclusion, it can be said that the congruence under consideration $x^3 \equiv b^3\ (mod\ 9m)$ has exactly three solutions given by $x \equiv 3mk + b\ (mod\ 9m)$ for $k = 0, 1, 2; m\ a\ positive\ integer$ and there are $\frac{1}{3}(9m) = 3m$ such solvable congruence.

Illustrations

Consider the congruence $x^3 \equiv 28\ (mod\ 81)$

It can be written as $x^3 \equiv 28 + 12.81 = 1000 = 10^3\ (mod\ 81)$ which is of the type $x^3 \equiv b^3\ (mod\ 9.9)$ (Roy, 2016).

Having solutions given by $x \equiv 3mk + b\ (mod\ 9.9)$ with $k = 0, 1, 2$.

Here we have $b = 10, m = 9$

Therefore, corresponding solutions are $x \equiv 3.9.k$ +10 (mod 81) with $k = 0, 1, 2$.

$$\equiv 27k + 10 \ (mod\ 81)\ with\ k = 0, 1, 2.$$
$$\equiv 10, 37, 64 \ (mod\ 81)$$

Therefore, required solutions are $x \equiv 10, 37, 64 \ (mod\ 81)$.
It is also found that the congruence has no other solutions.

Now consider a more general congruence $x^3 \equiv b^3 \ (mod\ 54)$
As m=6, hence there are $\frac{1}{3}(54) = 18$ such congruence each having exactly three solutions given by $x \equiv 3mk + b \ (mod\ 54)\ \ i.e. x \equiv 18k + b \ (mod\ 54) for\ k = 0, 1, 2$

Thus, all the 18-congruence with three solutions each are:

$$x^3 \equiv 1^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 1, 19, 37 \ (mod\ 54)$$
$$x^3 \equiv 2^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 2, 20, 38 \ (mod\ 54)$$
$$x^3 \equiv 3^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 3, 21, 39 \ (mod\ 54)$$
$$x^3 \equiv 4^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 4, 22, 40 \ (mod\ 54)$$
$$x^3 \equiv 5^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 5, 23, 41 (mod\ 54)$$
$$x^3 \equiv 6^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 6, 24, 42 \ (mod\ 54)$$
$$x^3 \equiv 7^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 7,25, 43 \ (mod\ 54)$$
$$x^3 \equiv 8^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 8, 26,44 (mod\ 54)$$
$$x^3 \equiv 9^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 9, 27,45 (mod\ 54)$$
$$x^3 \equiv 10^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 10, 28, 46 \ (mod\ 54)$$
$$x^3 \equiv 11^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 11, 29, 47 \ (mod\ 54)$$
$$x^3 \equiv 12^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 12, 30, 48 \ (mod\ 54)$$
$$x^3 \equiv 13^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 13, 31, 49 \ (mod\ 54)$$
$$x^3 \equiv 14^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 14, 32, 50 (mod\ 54)$$
$$x^3 \equiv 15^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 15, 33, 51 \ (mod\ 54$$
$$x^3 \equiv 16^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 16,34, 52 \ (mod\ 54)$$
$$x^3 \equiv 17^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 17, 35,53 (mod\ 54)$$
$$x^3 \equiv 18^3 \ (mod\ 54)\ \ with\ solutions\ x \equiv 18, 36,54 (mod\ 54)$$

Conclusion

Thus, we conclude that the congruence $x^3 \equiv b^3 \ (mod\ 9m)$, has exactly three incongruent solutions given by $x \equiv 3mk + b \ (mod\ 9m) for\ k = 0, 1, 2$ and for any positive integer m.

Merit of the Paper

Formulation of the solutions of the standard cubic congruence is the merit of the paper. A direct formula is established for the solutions. It makes the effort comfortable to find the solutions.

References

1. Roy, B.M. 2016. Discrete Mathematics and Number Theory, 1st Edition, Das Ganu Prakashan, Nagpur, India.

2. Thomas Koshy, 2007. Elementary Number Theory with Applications, 2nd Edition, Academic press.

3. Zuckerman *et al.*, 2008. An Introduction to the Theory of Numbers, Fifth edition, Wiley India Pvt. Ltd.