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FORMULATION OF SOLUTIONS OF TWO SPECIAL CLASSES OF CONGRUENCE OF COMPOSITE MODULUS OF HIGHER DEGREE

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Abstract: In this paper, solutions of two classes of congruence of composite modulus of higher degree of the type $x^{2n} \equiv a^{2n} \ (mod\ 8p^m)$, and $x^{2n+1} \equiv a^{2n+1} \ (mod\ 8p^m)\ with\ \ p$ an odd positive prime integer are formulated. It is also found that such congruence have eight and four solutions, respectively. No formula is found in the literature for solutions. Formulation is the merit of the paper.
Keywords: Binomial Expansion, Composite modulus, congruence of higher degree.

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Introduction

Number Theory is a neglected branch of mathematics. Some earlier Mathematicians had researched on it. Those are found in the literature. Nothing new is found. No one has even tried to read the branch today. In the books of Number Theory, a little is written about the standard congruence of composite modulus of higher degree. Only polynomial congruence of prime power modulus are found. This pained me very much. I am fond of number theory. To have a relief, I started research on this branch of mathematics. What I found, is presented in this paper.

The congruence $x^2 \equiv a \ (mod\ 2^n)\ with\ n \geq 3\ and\ a \equiv 1\ (mod\ 8)$ has exactly four incongruent solutions (Niven *et al.*, 2008).

Also, the congruence $x^2 \equiv a\ (mod\ p^m)$ with$\left(\frac{a}{p}\right) = 1$, has exactly two solutions (Niven *et al.*, 2008).

Then, abruptly a lighting-flash of the congruence $x^{2n} \equiv$
$a^{2n}(mod\ 2^3 . p^m)$……………………………(1)

and $x^{2n+1} \equiv a^{2n+1}\ (mod\ 2^3p^m)$……………………………..(2)

$with\ p\ an$ odd positive Prime integer is seen. I consider the problem for the solutions. No formula is found in the literature for solutions. So, I tried my best to formulate the congruence. The formulation is presented in this paper.

Problem-Statement

Here, the congruence $x^{2n} \equiv a^{2n}(mod\ 8.p^m)$, an odd positive prime; m, n any positive integer is to formulate. We show that the congruence $x^{2n} \equiv a^{2n}(mod\ 8.p^m)$, an odd positive prime; m, n any positive integer, has exactly eight solutions given by $x \equiv 8p^m \pm a;\ 6p^m \pm a;\ \ 4p^m \pm a;\ \ 2p^m \pm a\ (mod\ 8.p^m)$.

Also, the congruence $x^{2n+1} \equiv a^{2n+1}(mod\ 8.p^m)$ has only four solutions when a is an even integer.

These are given by $x \equiv 8p^m + a;\ 6p^m + a;\ \ 4p^m + a;\ \ 2p^m + a\ (mod\ 8.p^m)$.

Formulation

Consider the congruence (1). Such congruence are always solvable.

Here, $x^{2n} \equiv a^{2n}(mod\ 8.p^m)\ \ i.e.\ \ x^{2n} - a^{2n} \equiv 0\ (mod\ 8p^m)\ \ i.e.\ \ (x-a).f(x) \equiv 0(mod\ 8p^m)$

$i.e.\ \ x - a \equiv .(mod\ 8p^m)\ \ i.e.\ \ x \equiv a\ (mod\ 8p^m)$ is one of the solutions of the congruence.

Thus, two solutions are $x \equiv \pm a\ (mod\ 8p^m)$.

But if $the\ given\ congruence\ is\ x \equiv b\ (mod\ 8p^m)$ with $b \neq 8p^m$, then we add k.$8p^m\ to\ b\ so\ that$

$b + k.8p^m = a^{2n}, the\ given\ congruence\ becomes\ \ x^{2n} \equiv a^{2n}\ (mod\ 8p^m)$ and the solutions are as in above[1].

Now for other solutions, consider $x \equiv 4p^m \pm a$.

$$\text{Then, } x^{2n} = (4p^m \pm a)^{2n} = (4p^m)^{2n} + 2n.(4p^m)^{2n-1}.a + \cdots\ldots\ldots\ldots. + a^{2n}$$

$$= a^{2n} + 8p^m(\ldots\ldots\ldots.)$$

$$\equiv a^{2n}\ (mod\ 8p^m)$$

Thus, $x \equiv 4p^m \pm a\ (mod\ 8p^m)$ are the other two solutions.

$$\text{Similarly, } x^{2n} = (6p^m \pm a)^{2n} = (6p^m)^{2n} + 2n.(6p^m)^{2n-1}.a + \cdots\ldots\ldots\ldots. + a^{2n}$$

$$= a^{2n} + 8p^m(\ldots\ldots\ldots.)$$

$$\equiv a^{2n}\ (mod\ 8p^m)$$

Thus, $x \equiv 6p^m \pm a$ are the two other solutions.

Similarly, it can also be shown that $x \equiv 2p^m \pm a \ (mod\ 8p^m)$ are the other two solutions.

Therefore, all the eight solutions are given by

$$\boldsymbol{x \equiv 8p^m \pm a; 6p^m \pm a; 4p^m \pm a; 2p^m \pm a\ (mod\ 8p^m).}$$

Now we consider the congruence of the type (2).

AS it is the congruence of odd degree, hence$x = 8p^n - a, 6p^n - a,\ 4p^n - a,\ 2p^n - a$ cannot be the solutions of the congruence.

Thus, we can conclude that the only four solutions are

$$\boldsymbol{x \equiv 8p^n + a, 6p^n + a,\ 4p^n + a,\ 2p^n + a\ (mod\ 8p^n)}$$

Illustrations

The method is illustrated by giving two examples below.

Example-1] Consider the congruence $x^6 \equiv 24\ (mod\ 40)$.

It can be written as $x^6 \equiv 24 + 40 = 64 = 2^6\ (mod\ 2^3.5)[1]$

It is now of the type $x^{2n} \equiv a^{2n}(mod\ 2^3.p)\ \ with\ a = 2, p = 5.$

The corresponding solutions are given by

$$x \equiv 8p \pm a;\ \ 6p \pm a;\ \ 4p \pm a;\ \ 2p \pm a\ (mod\ 2^3.p)$$

$$i.e.\ \ x \equiv 8.5 \pm 2;\ \ 6.5 \pm 2;\ \ 4.5 \pm 2;\ \ 2.5 \pm 2\ (mod\ 2^3.5)$$

$$i.e.\ \ x \equiv 40 \pm 2; 30 \pm 2; 20 \pm 2; 10 \pm 2\ \ (mod\ 40)$$

$$i.e.\ \ x \equiv\ \ 2, 38; 28, 32;\ \ 18, 22;\ \ 8, 12\ (mod\ 40)$$

$$i.e.\ \ x \equiv 2, 8, 12, 18, 22, 28, 32, 38\ (mod\ 40).$$

Example-2: Consider the congruence $x^4 \equiv 9\ (mod\ 72)$.

It can be written as $x^4 \equiv 9 + 72 = 81 = 3^4\ (mod\ 8.9)$

It is now of the type $x^{2n} \equiv a^{2n}(mod\ 8.p^2)\ \ with\ a = 2, p = 3.$

The corresponding solutions are given by

$$x \equiv 8p^2 \pm a;\ \ 6p^2 \pm a;\ \ 4p^2 \pm a;\ \ 2p^2 \pm a\ (mod\ 2^3.3^2)$$

$$i.e.\ \ x \equiv 8.9 \pm 3;\ \ 6.9 \pm 3;\ \ 4.9 \pm 3;\ \ 2.9 \pm 3\ (mod\ 2^3.9)$$

$$i.e.\ \ x \equiv 72 \pm 3; 54 \pm 3; 36 \pm 3; 18 \pm 3\ \ (mod\ 72)$$

$$i.e.\ \ x \equiv\ \ 3, 69;\ 51, 57; 33, 39; 15, 21\ (mod\ 72)$$

Example-3: Consider the example $x^5 \equiv 16\ (mod\ 72)$.

It can be written as $x^5 \equiv 16 + 1008 = 1024 = 4^5 \ (mod\ 8.9)$

It is now of the type $x^{2n+1} \equiv a^{2n+1}(mod\ 8.p^2)\ \ with\ a = 4, an\ even\ integer;\ p = 3.$

The corresponding solutions are given by

$$x \equiv 8p^2 + a;\ \ 6p^2 + a;\ \ 4p^2 + a;\ \ 2p^2 + a\ (mod\ 2^3.3^2)$$

$$i.e.\ \ x \equiv 8.9 + 4;\ \ 6.9 + 4;\ \ 4.9 + 4;\ \ 2.9 + 4\ (mod\ 2^3.9)$$

$$i.e.\ \ x \equiv 72 + 4; 54 + 4; 36 + 4; 18 + 4\ \ (mod\ 72)$$

$$i.e.\ \ x \equiv\ \ 4, 58, 40, 22\ (mod\ 72)$$

Example-4: Consider the congruence $x^5 \equiv 32\ (mod\ 88)$

It can be written as $x^5 \equiv 2^5\ (mod\ 8.11)$

It is now of the type $x^{2n+1} \equiv a^{2n+1}(mod\ 8.p)\ \ with\ a = 2, an\ even\ integer;\ p = 11.$

The corresponding solutions are given by

$$x \equiv 8p + a;\ \ 6p + a;\ \ 4p + a;\ \ 2p + a\ (mod\ 8.11)$$

$$i.e.\ \ x \equiv 8.11 + 2;\ \ 6.11 + 2;\ \ 4.11 + 2;\ \ 2.11 + 2\ (mod\ 8.11)$$

$$i.e.\ \ x \equiv 88 + 2; 66 + 2; 44 + 2; 22 + 2\ \ (mod\ 8.11)$$

$$i.e.\ \ x \equiv\ \ 2, 68, 46, 24\ (mod\ 88)$$

$$i.e.\ x \equiv 2, 24, 46, 68\ (mod\ 88)$$

Conclusion

Thus, it can be said that the congruence $x^{2n} \equiv\ a^{2n}(mod\ 8.p^m)$, p an odd positive prime integer has exactly eight solutions given by $x \equiv 8p^m \pm a;\ \ 6p^m \pm a;\ \ 4p^m \pm a;\ \ 2p^m \pm a\ (mod\ 2^3.p^m)$. While the congruence $x^{2n+1} \equiv a^{2n+1}\ (mod\ 8p^m)$ has only four solutions given by

$$\boldsymbol{x \equiv 8p^n + a, 6p^n + a,\ \ 4p^n + a,\ \ 2p^n + a\ (mod\ 8p^n)}$$ **, if a is an even integer.**

Merit of the Paper

In this paper, two classes of standard congruence of higher degree of composite modulus is formulated. No method is found in the literature for the solutions. Even no formula was established for the solutions of the congruence. Now, formulation is done. This is the merit of the paper.

References

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